CHARLES DARWIN UNIVERSITY

Family Name	
Given Names	
Student Number	
Teaching Period	Semester 2, 2016

FINAL EXAMINATION	DURATION
SMA211 – Mathematics 2b	Reading Time: **10** minutes Writing Time: **180** minutes

INSTRUCTIONS TO CANDIDATES

1.1 This paper contains six questions. Answer all Six (6) questions.

1.2 All questions are of equal value, and parts carry marks as indicated.

1.3 All symbols, unless stated otherwise, have their usual meanings.

1.4 Read ALL questions carefully.

1.5 Answers without showing detailed working will attract little marks.

EXAM CONDITIONS

<u>You may begin writing from the commencement of the examination session.</u> The reading time indicated above is provided as a guide only.

This is a CLOSED BOOK examination

Any non-programmable calculator is permitted

No handwritten notes are permitted

No dictionaries are permitted

ADDITIONAL AUTHORISED MATERIALS	EXAMINATION MATERIALS TO BE SUPPLIED
No additional printed material is permitted	1 x 20 Page Book 1 x Scrap Paper Formula Sheet/s

THIS EXAMINATION IS PRINTED DOUBLE-SIDED.

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Question 1

(a) Find the Laplace transform of $f(t) = 2\sin(3t - \pi/2)$. Explain the steps used in determining it. **(Marks 7)**

(b) Find the inverse Laplace transform of the following:

$$F(s) = \frac{\pi}{s^2 + 10\pi s + 24\pi^2}$$ **(Marks 6)**

(c) Using the theorem of determining the Laplace transform of the integral of a function, find the inverse Laplace transform of the following: $F(s)$:

$$F(s) = \frac{1}{s(s^2 + \omega^2)}.$$ **(Marks 7)**

Question 2

(a) Using the method of Laplace transform, solve the following initial value problem:

$$y'' + 7y' + 12y = 21e^{3t}, \; y(0) = 0, \quad y'(0) = 0.$$ **(Marks 7)**

(b) Find the inverse Laplace transform of the following $F(s)$:

$$F(s) = \frac{e^{-s}}{s^2 + \pi^2} + \frac{e^{-2s}}{s^2 + 4} + \frac{e^{-3s}}{(s+2)^2}.$$ **(Marks 6)**

(c) Using the Laplace transform method, solve the following equation:

$$y(t) + 2e^t \int_0^t y(t')e^{-t'} dt' = te^t \; .$$ **(Marks 7)**

Question 3

(a) Find the Laplace transform of $f(t) = \dfrac{1}{2\beta^3}\left(\sin\beta t - \beta t\cos\beta t\right)$. Express it in the simplest form. **(Marks 6)**

(b) By differentiating the function $f(t) = \dfrac{1}{2}te^{-3t}$, find its Laplace transform. **(Marks 6)**

(c) Using the method of Laplace transform, solve the following set of first order ODEs.

$$\begin{aligned} y_1' &= 4y_1 + y_2 \\ y_2' &= -y_1 + 2y_2 \end{aligned}, \quad y_1(0) = 0,\, y_2(0) = 1.$$ **(Marks 8)**

Question 4

(a) Evaluate the following line integral anticlockwise

$$\oint_C \frac{dz}{z^2+4},$$

along the ellipse C given by $4x^2 + (y-2)^2 = 4$. **(Marks 7)**

(b) Using the Cauchy's integral formula, evaluate the following contour integral:

$$\oint_C \frac{\sin z}{z+2iz}dz$$

clockwise along the curve $C : |z-4-2i| = 5$. **(Marks 6)**

(c) Evaluate the following line integral

$$\oint_C \frac{(1+z)\sin z}{(2z-1)^2}dz$$

anticlockwise along the curve $C : |z-i| = 2$. **(Marks 7)**

Question 5

(a) The real part of a complex function $f(z)$ is given by $u(x,y) = ax^3 + bxy$.

(i) Determine the values of a and b for which this function will be harmonic. **(Marks 4)**

(ii) Find the harmonic conjugate $v(x,y)$ of the above $u(x,y)$ and then find the complex function $f(z)$. **(Marks 6)**

(b) The efficiency (in %) of seven Francis turbines are measured as given below:

91.8 89.1 89.9 92.5 90.7 91.2 91.0 .

Represent these efficiency data by a box plot. **(Marks 5)**

(c) A box contains 10 screws, three of which are defective. Two screws are drawn at random with and without the replacement. In each case, find the probability that neither of the two screws is defective. **(Marks 5)**

Question 6

(a) How many automobile registrations may the police have to check in a hit-and-run accident if a witness reports KDP7 and cannot remember the last two digits on the licence plate but is certain that all three were different numbers? **(Marks 5)**

(b) If the life of ball bearings has the density $f(x) = ke^{-0.2x}$ for $0 \le x \le 10$ and 0 otherwise, what is the value of k? Find the probability $P(X \ge 5)$. **(Marks 5)**

(c) Find the maximum likelihood estimate of θ in the density function $f(x) = \theta e^{-\theta x}$ if $x \ge 0$ and $f(x) = 0$ if $x < 0$. **(Marks 5)**

(d) A sample of lengths of 20 bolts with mean 15.50 cm and variance 0.09 cm^2 is drawn from a normal population, determine a 99% confidence interval for the mean μ of the population. **(Marks 5)**

SMA211 : Final Exam - 2016

Guiding Solutions

1 (a) $f(t) = 2\sin(3t - \pi/2)$

$= 2[\sin 3t \cos \pi/2 - \cos 3t \sin \pi/2]$

$\cos \pi/2 = 0$

$= 2[-\cos 3t \cdot 1] = -2\cos 3t$

$\therefore \mathcal{L}[f(t)] = -2\,\mathcal{L}[\cos 3t]$

$= -2\dfrac{s}{s^2+3^2} = -\dfrac{2s}{s^2+9}$

(b) $F(s) = \dfrac{\pi}{s^2+10\pi s+24\pi^2}$

$= \dfrac{\pi}{s^2+10\pi s+25\pi^2-\pi^2}$

$= \dfrac{\pi}{(s+5\pi)^2 \cancel{+} - \pi^2}$

using $\mathcal{L}^{-1}\left[F(s) = \dfrac{a}{s^2-a^2}\right] = \sinh at$

$\mathcal{L}^{-1}\left[F(s) = \dfrac{\pi}{(s+5\pi)^2-\pi^2}\right] = e^{-5\pi t}\sinh \pi t$

1(c) Using $\mathcal{L}\left[\int_0^t f(\tau)\,d\tau\right] = \dfrac{F(s)}{s}$

Given that

$$F(s) = \frac{1}{s(s^2+\omega^2)}$$

$$\mathcal{L}[\sin \omega t] = \frac{\omega}{s^2+\omega^2}$$

$$\therefore \mathcal{L}^{-1}\left[\frac{1}{s^2+\omega^2}\right] = \frac{1}{\omega}\sin \omega t$$

This can be used to calculate:

$$\mathcal{L}^{-1}\left[\frac{1}{s(s^2+\omega^2)}\right] = \frac{1}{\omega}\int_0^t \sin \omega\tau \, d\tau$$

$$= \frac{1}{\omega}\left[-\frac{\cos \omega\tau}{\omega}\right]_0^t$$

$$= \frac{1}{\omega^2}\left[-\cos \omega t + 1\right]$$

$$= \frac{1}{\omega^2}\left[1 - \cos \omega t\right]$$

2(a) $y'' + 7y' + 12y = 21e^{3t} \qquad y'(0) = y(0) = 0$

Take Laplace transform of both sides:

$$\mathcal{L}[y''] + 7\mathcal{L}[y'] + 12\mathcal{L}(y) = 21\mathcal{L}(e^{3t})$$

$$s^2Y(s) - y'(0) - s\,y(0) + 7[sY(s) - y(0)] + 12Y(s) = \frac{21}{s-3}$$

$$\Rightarrow (s^2 + 7s + 12)Y(s) = \frac{21}{s-3}$$

$$\Rightarrow (\cancel{s^2}+3)(s+4)\,Y(s) = \frac{21}{s-3}$$

$$\Rightarrow Y(s) = \frac{21}{(s-3)(\cancel{s}+3)(s+4)}$$

$$= \frac{21}{(\cancel{s}-3)}\left[\frac{1}{s+3} - \frac{1}{s+4}\right]$$

$$= 21\left[\frac{1}{(s-3)(s+3)} - \frac{1}{(s-3)(s+4)}\right]$$

$$= 21\left[\frac{1}{s^2-3^2} - \frac{1}{7}\left\{\frac{1}{s-3} - \frac{1}{s+4}\right\}\right]$$

Take the inverse L.T.

$$\mathcal{L}^{-1}[Y(s)] = 21\left[\mathcal{L}^{-1}\left(\frac{1}{(s^2-3^2)}\right) - \frac{1}{7}\left\{\mathcal{L}^{-1}\left[\frac{1}{s-3}\right] - \mathcal{L}^{-1}\left[\frac{1}{s+4}\right]\right\}\right]$$

$$y(t) = 21\left[\frac{\sinh 3t}{3} - \frac{1}{7}\left\{e^{3t} - e^{4t}\right\}\right]$$

$$= 7\sinh 3t - 3\left\{e^{3t} - e^{4t}\right\}$$

$$\boxed{y(t) = 7\sinh 3t - 3e^{3t} + 3e^{4t}}$$

2 (b) $$F(s) = \frac{e^{-s}}{s^2+\pi^2} + \frac{e^{-2s}}{s^2+4} + \frac{e^{-3s}}{(s+2)^2}$$

Use

$$\mathcal{L}[f(t-a)u(t-a)] = e^{-as}F(s)$$

$$\Rightarrow \mathcal{L}^{-1}[e^{-as}F(s)] = f(t-a)\,u(t-a)$$

$$\mathcal{L}^{-1}[F(s)] = \mathcal{L}^{-1}\left[\frac{e^{-s}}{s^2+\pi^2}\right] + \mathcal{L}^{-1}\left[\frac{e^{-2s}}{s^2+4}\right] + \mathcal{L}^{-1}\left[\frac{e^{-3s}}{(s+2)^2}\right]$$

$$f(t) = \frac{1}{\pi}\sin\pi(t-1)\,u(t-1) + \frac{1}{2}\sin 2(t-2)\,u(t-2) + (t-3)\,u(t-3)\,e^{-2(t-3)}$$

$$= \frac{1}{\pi}\sin\pi(t-1)\,u(t-1) + \frac{1}{2}\sin 2(t-2)\,u(t-2) + (t-3)\,e^{-2(t-3)}\,u(t-3).$$

2(c) $\quad y(t) + 2e^{t}\int^{t} y(t')e^{-t'}dt' = te^{t}$

This can be written as :

$$\Rightarrow y(t) + 2\int_0^t y(t')\, e^{+(t-t')}\, dt' = te^{t}$$

Take L.T. both sides :

$$\mathcal{L}[y(t)] + 2\mathcal{L}\left[\int_0^t y(t')\, e^{+(t-t')}\, dt'\right] = \mathcal{L}(te^{t})$$

$$\Rightarrow Y(s) + \frac{2Y(s)}{s-1} = \frac{1}{(s-1)^2}$$

$$\Rightarrow Y(s)\left[1 + \frac{2}{s-1}\right] = \frac{1}{(s-1)^2}$$

$$\Rightarrow Y(s)\left[\frac{s+1}{s-1}\right] = \frac{1}{(s-1)^2}$$

$$\Rightarrow Y(s) = \frac{\cancel{(s-1)}}{(s+1)(s-1)^{\cancel{2}}} = \frac{1}{(s-1)(s+1)}$$

$$= \frac{1}{s^2-1}$$

Take inverse L.T.

$$y(t) = \mathcal{L}^{-1}\left[\frac{1}{s^2-1}\right] = \sinh t$$

3 (a) $$f(t) = \frac{1}{2\beta^3}(\sin\beta t - \beta t\cos\beta t)$$

$$\mathcal{L}[f(t)] = \frac{1}{2\beta^3}\left[\mathcal{L}[\sin\beta t] - \beta\mathcal{L}(t\cos\beta t)\right]$$

$$\mathcal{L}(\sin\beta t) = \frac{\beta}{s^2+\beta^2}$$

$$\mathcal{L}(t\cos\beta t) = -F'(s) \qquad \mathcal{L}[\cos\beta t] = \frac{s}{s^2+\beta^2}$$

$$\therefore \mathcal{L}(t\beta t] = -\frac{d}{ds}\left[\frac{s}{s^2+\beta^2}\right]$$

$$= -\left[\frac{1}{(s^2+\beta^2)} - s(s^2+\beta^2)^{-2}\times 2s\right.$$

$$= -\left[\frac{1}{s^2+\beta^2} - \frac{2s^2}{(s^2+\beta^2)^2}\right.$$

$$= -\left[\frac{\cancel{s^2}+\beta^2-\cancel{2}s^2}{(s^2+\beta^2)^2}\right] = \frac{-(\beta^2-s^2)}{(s^2+\beta^2)^2}$$

$$= \frac{s^2-\beta^2}{(s^2+\beta^2)^2}$$

Substituting above

$$\mathcal{L}(f(t)) = \frac{1}{2\beta^3}\left[\frac{\beta}{s^2+\beta^2} - \frac{\beta(s^2-\beta^2)}{(s^2+\beta^2)^2}\right]$$

$$= \frac{1}{2\beta^3}\left[\beta(s^2+\beta^2) - \beta(s^2-\beta^2)\right]\Big/(s^2+\beta^2)^2$$

$$\rightarrow = \frac{1}{(s^2+\beta^2)^2} \qquad = \frac{\beta}{2\beta^3}\left[\frac{\cancel{s^2}+\beta^2-\cancel{s^2}+\beta^2}{(s^2+\beta^2)^2}\right] = \frac{\cancel{2\beta^2}}{\cancel{2\beta^2}(s^2+\beta^2)^2}$$

3(b) $f(t) = \frac{1}{2} t e^{-3t}$

Take derivative both side:

$$f'(t) = \frac{1}{2} e^{-3t} + \frac{1}{2} t \times e^{-3t} \times -3$$

$$= \frac{1}{2} e^{-3t} - \frac{3}{2} t e^{-3t}$$

Take L.T. both sides:

$$\mathcal{L}[f'(t)) = \frac{1}{2} \mathcal{L}[e^{-3t}] - 3 \mathcal{L}(\tfrac{1}{2} t e^{-3t})$$

$$sF(s) - f(0) = \frac{1}{2} \frac{1}{s+3} - 3F(s)$$

$$\Rightarrow (s+3)F(s) = \frac{1}{2} \frac{1}{s+3} \qquad (f(0) = 0)$$

$$\Rightarrow F(s) = \frac{1}{2} \frac{1}{(s+3)^2}$$

3(c)

$$y_1' = 4y_1 + y_2 \qquad y_1(0) = 0 \; \cancel{y_2(0) = 0} \; y_2(0) = 1$$

$$y_2' = -y_1 + 2y_2$$

Take L.T. both sides

$$\mathcal{L}(y_1') = 4\mathcal{L}(y_1) + \mathcal{L}(y_2)$$

$$\mathcal{L}(y_2') = -\mathcal{L}(y_1) + 2\mathcal{L}(y_2)$$

Using $\mathcal{L}(y') = sY(s) - sy(0) - y'(0)$

We get:

$$SY_1 - sy_1(0) - y'(0) = 4Y_1(s) + 2Y_2(s)$$

$$SY_2 - sy_2(0) - y_2'(0) = -Y_1(s) + 2Y_2(s)$$

$$\Rightarrow \quad SY_1 - 4Y_1(s) - Y_2(s) = 0$$

$$Y_1(s) + (S-2)Y_2(s) = 1$$

$$\Rightarrow \quad (S-4)Y_1(s) - Y_2(s) = 0 \quad \text{——— (1)}$$

$$Y_1(s) + (S-2)Y_2(s) = 1 \quad \text{——— (2)}$$

Multiply (1) by $(S-2)$:

$$(S-4)(S-2)Y_1(s) - (S-2)Y_2(s) = 0 \quad \text{——— (3)}$$

$$Y_1(s) + (S-2)Y_2(s) = 1 \quad \text{——— (4)}$$

Add (3) and (4)

$$[(s-4)(s-2)+1]\,Y_1(s) = 1$$

$$Y_1(s) = \frac{1}{(s-4)(s-2)+1} = \frac{1}{s^2-6s+8+1}$$

$$= \frac{1}{(s-3)^2} \qquad \text{——— (5)}$$

From (1)

$$Y_2(s) = (s-4)\,Y_1(s) = \frac{(s-4)}{(s-3)^2}$$

$$= \frac{(s-3)-1}{(s-3)^2} = \frac{1}{(s-3)} - \frac{1}{(s-3)^2}$$

Take inverse L.T.

$$\mathcal{L}^{-1}[Y_1(s)] = \mathcal{L}^{-1}\left[\frac{1}{(s-3)^2}\right] \Rightarrow y_1(t) = t\,e^{3t}$$

$$\mathcal{L}^{-1}[Y_2(s)] = \mathcal{L}^{-1}\left[\frac{1}{s-3}\right] - \mathcal{L}^{-1}\left[\frac{1}{(s-3)^2}\right]$$

$$\Rightarrow y_2(t) = e^{3t} - t\,e^{3t}$$

$$\therefore \boxed{\begin{aligned} y_1(t) &= t\,e^{3t} \\ y_2(t) &= e^{3t} - t\,e^{3t} \end{aligned}}$$

4 (a) $\oint_C \frac{dz}{z^2+4}$

C : An ellipse of eqn. $4x^2 + (y-2)^2 = 4$

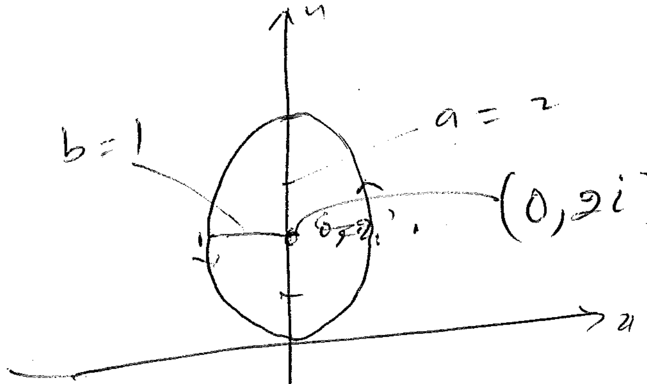
The Centre of the ellipse is at $(0,2)$.

Dividing the above eqn by 4 we get the standard eqn for an ellipse as

$$\frac{\cancel{4}x^2}{\cancel{4}} + \frac{(y-2)^2}{4} = \frac{\cancel{4}}{\cancel{4}},$$

$$= x^2 + \frac{(y-2)^2}{2^2} = 1$$

Here semi axes of ellipse are $a = 1$ $b = 2$,



The integral in $\oint_C \frac{dz}{z^2+4}$ has discontinuity

at $z = \pm 2i$, $z = 2i$ is within the ellipse.

Therefore writing $\int \frac{dz}{(z+2i)(z-2i)}$

$$\oint_C \frac{\left(\frac{1}{z+2i}\right) dz}{(z-2i)} = 2\pi i \left(\frac{1}{2i+2i}\right)$$

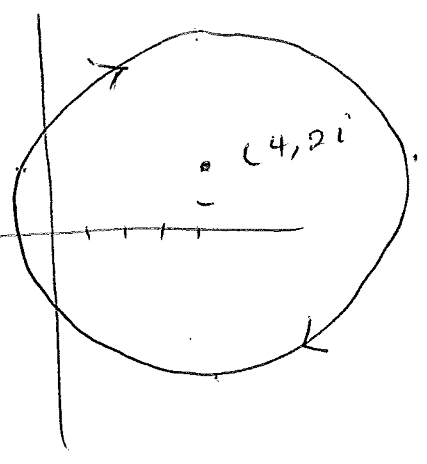
$$= \frac{2\pi \cancel{i}}{\cancel{4}_{2} \cancel{i}} = \pi/2$$

4(b) $\oint_C \frac{\sin z \, dz}{z+2iz}$ C: Clockwise circle $|z-4-2i| = 5$

~~circle of centre is~~
Centre of circle is at $z = 4+2i$ and radius 5

The above integral can be written as

$$\oint_C \frac{\sin z \, dz}{z(z+2i)}$$



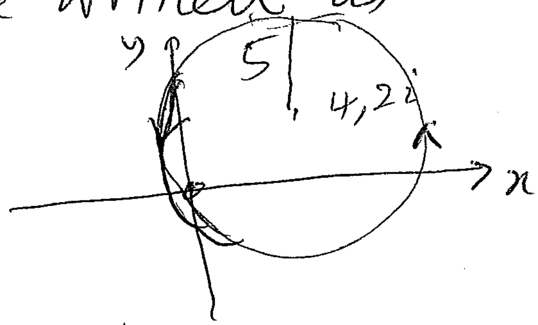
The integrand has discontinuities at $z=0$ and $z=-2i$ ~~both of which fall within the circle.~~

~~The above integral can be factorised as~~

~~$\oint \frac{\sin z}{2i}\left[\frac{1}{z} - \frac{1}{z+2i}\right] dz$~~

4(b) $\oint_C \frac{\sin z \, dz}{z + 2iz}$ C: clockwise circle $|z-4-2i|=5$

The integral can be written as



$$\frac{1}{(1+2i)} \oint_C \frac{\sin z \, dz}{z}$$

The integrand has discontinuity at $z = 0$, which is within the given circle.

$$\therefore \frac{1}{(1+2i)} \oint_C \frac{\sin z}{z} dz = -2\pi i \frac{\sin(0)}{(1+2i)}$$

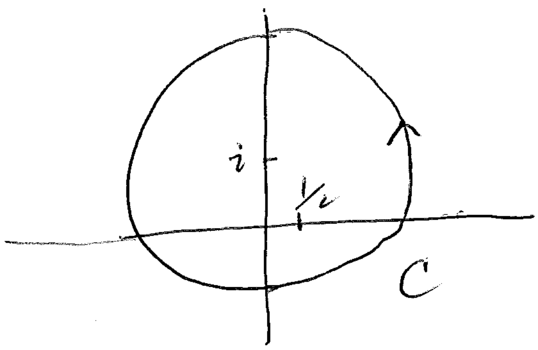
$$= 0.$$

4(c) $\oint_C \frac{(1+z)\sin z}{(2z-1)^2} dz \qquad C: |z-i| = 2$

$$= \oint_C \frac{(1+z)\sin z}{4(z-1/2)^2} dz \qquad \text{——— (1)}$$

This is of the form of

$$f'(z_0) = \frac{1}{2\pi i}\int \frac{f(z)\,dz}{(z-z_0)^2} \qquad \text{——— (2)}$$

C: 

$z = 1/2$ is within the contour.

Using (2), (1) can be written as.

$$\oint_C \frac{\left[\frac{(1+z)\sin z}{4}\right]}{(z-1/2)^2} dz \equiv \oint \frac{f(z)\,dz}{(z-1/2)^2}$$

where $f(z) = \frac{(1+z)\sin z}{4}$

$$f'(z) = \frac{\sin z + (1+z)\cos z}{4}$$

$$f'(1/2) = \frac{\sin(1/2) + (1+1/2)\cos 1/2}{4}$$

$$f'(1/2) = \frac{\sin(0.5) + \frac{3}{2}\cos(0.5)}{4}.$$

$$\therefore \oint_C \frac{(1+z)\sin z}{4(z-1/2)^2}\,dz = 2\pi i\left[\frac{\sin(0.5)+\frac{3}{2}\cos(0.5)}{4}\right]$$

$$= \frac{1}{2}\pi i\left[\sin(0.5) + \frac{3}{2}\cos(0.5)\right]$$

5 (a) (i) $u(x, y) = ax^3 + bxy$

If it is harmonic it should satisfy Laplace's equation.

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

$$\frac{\partial u}{\partial x} = 3ax^2 + by \qquad \frac{\partial u}{\partial y} = bx$$

$$\frac{\partial^2 u}{\partial x^2} = 6ax \qquad \frac{\partial^2 u}{\partial y^2} = 0$$

For $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$, $\quad 6ax = 0$

$\therefore \quad a = 0$

(ii) $u(x, y) = bxy$

Use Cauchy-Riemann eqns to determine v

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad -(1) \qquad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x} \quad -(2)$$

From (1)

$$\frac{\partial u}{\partial x} = by = \frac{\partial v}{\partial y}$$

$$\frac{\partial v}{\partial y} = by$$

$$v = \int by\,dy + k(x)$$

$$= \frac{1}{2}by^2 + k(x)$$

Using (2)

$$\frac{\partial v}{\partial x} = k'(x) = -\frac{\partial u}{\partial y} = -bx$$

$$\therefore k(x) = -\int bx\,dx + c = -\frac{1}{2}bx^2 + c$$

$$v(x,y) = \frac{1}{2}by^2 - \frac{1}{2}bx^2 + c$$

Now $f(z) = u(x,y) + i\,v(x,y)$

$$= bxy + i[\tfrac{1}{2}by^2 - \tfrac{1}{2}bx^2 + c]$$

$$= bxy + ib[\tfrac{1}{2}y^2 + \tfrac{1}{2}(ix)^2] + c$$

$$= bxy + ib[\tfrac{1}{2}y^2 + \tfrac{1}{2}(ix)^2] + c_1$$

$$= ib[-ixy + \tfrac{1}{2}y^2 + \tfrac{1}{2}(ix)^2] + c_1$$

$$= ib[\tfrac{1}{2}(ix - y)^2] + c_1$$

$$= ib[\tfrac{i^2}{2}(x+iy)^2] + c_1$$

$$= -\frac{ib}{2}z^2 + C$$

5 (b) 91.8, 89.1, 89.9, 92.5, 90.7, 91.2, 91.0

Arrange in order

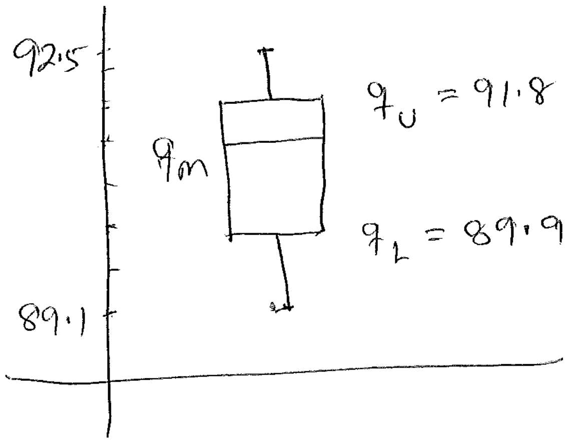
89.1, 89.9, 90.7, 91.0, 91.2, 91.8, 92.5

$R = x_{max} - x_{min} = 92.5 - 89.1 = 3.4$

Median 91.0

$q_L = 89.9$ $\quad$ $q_U = 91.8$

Interquartile range : $q_U - q_L = 91.8 - 89.9 = 1.9$



5(c) <u>with replacement</u>

We consider events A : First drawn screw is nondefective
B : second " " "

$\cancel{P(A)}$ 10 Screws 7 nondefective 3 defective

$$P(A) = \frac{7}{10} \quad P(B) = \frac{7}{10}$$

The events are independent

$$P(A \cap B) = \cancel{7}\, P(A)\, P(B)$$
$$= \frac{7}{10} \cdot \frac{7}{10} = 0.49 \text{ or } 49\%$$

without replacement

$$P(A) = \frac{7}{10} \quad P(B) = \frac{\cancel{6}\,\cancel{9}}{\cancel{9}\,3} = \frac{2}{3}$$

$$P(A \cap B) = P(A)\,P(B) = \frac{7}{10} \cdot \frac{2}{3} = 0.47$$
$$= 47\%.$$

6 (a) KDP 7$____1$ $\_\_\__2$

Last two digits unknown but all three digits were different.

$\_\_\__1$ can be 0, 1, 2, 3, 4, 5, 6, 8, or 9

$\therefore$ 9 possibilities

$\_\_\__2$ has 8 possibilities as it can't be the digit 7 or the digit for $\_\_\__1$.

Therefore $9 \times 8 = 72$ automobile registrations to be checked.

6 (b) $\int_{-\infty}^{\infty} f(x)dx = 1$

In this case

$$\int_0^{10} ke^{-0.2x}\, dx = 1$$

$$= k\left[\frac{e^{-0.2x}}{-0.2}\right]_0^{10}$$

$$= -5k\left[e^{-2} - 1\right]$$

$$\approx -5k[-0.8647] = 1$$

$$\therefore\ k \approx 0.2313$$

$$P(X \geq 5) = k\int_5^{10} e^{-0.2x}\, dx$$

$$= k\left[\frac{e^{-0.2x}}{-0.2}\right]_5^{10}$$

$$= -5k\left[e^{-2} - e^{-1}\right]$$

$$\approx -5k[-0.2325]$$

$\therefore = 0.269$ or 27%

6 (c) Given $f(x) = \theta e^{-\theta x}$ for $x \geq 0$

$= 0$ $x < 0$

Take discrete random variable x_j such that $x_j > 0$ and

$f(x_j) = \theta e^{-\theta x_j}$ for $j = 0, 1, 2, \cdots$

$= 0$ $x_j < 0$

Then we get

$$l = f(x_1)f(x_2)\cdots f(x_n)$$

$$= \theta^n e^{-\theta \sum_j x_j}$$

$$\ln|l| = n\ln|\theta| - \theta\sum_j x_j$$

$$\frac{\partial \ln|l|}{\partial\theta} = \frac{n}{\theta} - \sum_j x_j = 0$$

$$\therefore \frac{n}{\hat{\theta}} = \sum_j x_j$$

$$\therefore \hat{\theta} = \frac{n}{\sum_j x_j}$$

6 (d) $\gamma = 0.99$

$\therefore c = 2.576$

$\bar{x} = 15.50$ cm

Sample size $n = 20$

Variance, $\sigma^2 = 0.09\ \text{cm}^2$

$\therefore$ Standard deviation, $\sigma = 0.3$ cm

$$k = \frac{c\sigma}{\sqrt{n}} = \frac{2.576 \times 0.3}{\sqrt{20}} = 0.173$$

The confidence interval for μ is $\text{CONF}_{0.99}\{15.33 \le \mu \le 15.67\}$